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## Granular Material Behaviour under Dynamic Excitation

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# GRANULAR MATERIAL BEHAVIOUR UNDER DYNAMIC EXCITATION

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## ABSTRACT

This paper deals with the investigation of bulk material filled silos under seismic excitation. The described numerical model for a silo consists of three components, namely the granular material, an interface element between the granular material and the silo wall and the silo itself. The bulk material behaviour is described in four different ways: by the classical hypoplasticity theory, by two special versions of it which use time history functions and finally by the intergranular strain approach. The dynamic behaviour as described by these four different material laws is presented, as well as comparisons of the numerical results with experimental data.

## INTRODUCTION

The design of granular material silos subject to seismic excitation must guarantee the stability of the silo in the case of an earthquake in order to avoid disruptions in the flow of important materials for aiding the population and also for providing materials needed for the rebuilding process. In general, most of the current codes for silos under earthquake excitation are quite conservative so that an economically attractive design is not possible.

For the overall modelling of the behaviour of silos under seismic excitation, a realistic description of the dynamic behaviour of the granular material is essential. The contact problem between the granular material and the silo wall and the non-linear material behaviour of the granular material itself leads to a highly non-linear interaction problem. Linear models have been shown to be inadequate for a reliable solution of the overall problem.

The basic numerical model for the investigation of the dynamic behaviour of the granular material and the silo consists of the three-dimensional structure with the granular material, the contact area between the granular material and the silo shell, and the silo shell itself.

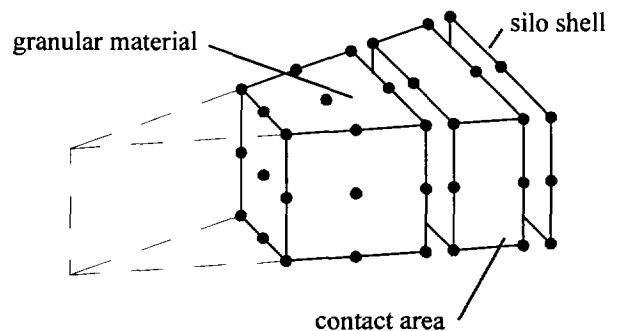


Figure 1: Discretisation of the silo

The silo shell is modelled as a 9-node shell element. The contact area is simulated with an 18-node interface element. A 27-node volume element is used for the granular material. It describes the material behaviour by using a hypoplastic material law which may be modified by applying one of two history functions; alternatively, the intergranular strain theory may be employed. The dynamic behaviour of these four different material laws will be evaluated by comparing their performance in simulating classical soil mechanics tests. Finally, filled silos will be modelled in order to check if the numerical model used is also able to simulate the behaviour of the whole structure.

## HYPOPLASTICITY

The material law employed for the granular material is the hypoplasticity theory which describes the material behaviour without using a yield surface or a plastic potential. It also does not distinguish between elastic and plastic deformation. The

hypoplasticity theory describes the stress rate  $\dot{\mathbf{T}}$  due to the strain-rate  $\mathbf{D}$ , the effective stress  $\mathbf{T}$  and a linear and non-linear stiffness-matrix  $\mathbf{L}$  and  $\mathbf{N}$  by the expression:

$$\dot{\mathbf{T}} = \mathcal{L}(\hat{\mathbf{T}}) : \mathbf{D} + \mathbf{N}(\hat{\mathbf{T}}) \|\mathbf{D}\| \quad (1)$$

In the following, four different versions of hypoplasticity will be described, namely the classical hypoplasticity theory, two special versions of it using time history functions, and finally the intergranular strain approach.

A comprehensive introduction is given by Kolymbas [2000]. The modified hypoplasticity theory of von Wolffersdorff [1996] is the base of the following investigation.

### Hypoplasticity modified with time history functions

It may be stated that the hypoplasticity theory describes the loading process of granular material adequately ; it is being widely used for numerical simulations for example in soil mechanics and in mechanical engineering. However the original version by von Wolffersdorff describes the energy dissipation during cyclic excitation only very approximately. The material law cannot describe the stiffness increase at the beginning of the reloading process. In contrary to the results predicted by this theory, the stiffness does not only depend on the effective stress but also on the stress time history. In order to overcome this defect, Bauer [1992] included a function describing the time dependent behaviour of granular material:

$$\dot{\mathbf{T}} = \mathcal{L}(\hat{\mathbf{T}}) : \mathbf{D} + \mathbf{N}(\hat{\mathbf{T}}) \|\mathbf{D}\| [1 + f_t] [1 + f_m] \quad (2)$$

The function  $f_t$  limits the compressibility by means of:

$$f_t = r_0 \exp[-r_1 I_{cr}] = r_0 \exp\left[-r_1 \frac{e - e_{min}}{e_{cr} - e_{min}}\right] \quad (3)$$

Here,  $r_0$  and  $r_1$  are material parameters and  $e_{cr}$ ,  $e_{min}$  and  $e$  are the critical, the minimal and the effective void ratio, respectively.

Bauer formulates the time history function  $f_m$  as a function of the displacement capacity:

$$f_m = \mu_1 \frac{A}{A_{max}} \frac{|\text{tr}(\mathbf{TD})| + \text{tr}(\mathbf{TD})}{\|\mathbf{T}\| \|\mathbf{D}\|} \quad (4)$$

Here, the history parameter  $A$  depends on the load path.

$$\dot{A} = \mu_2 [|\text{tr}(\mathbf{TD})| - \text{tr}(\mathbf{TD})] - \mu_3 \frac{A}{A_{max}} [|\text{tr}(\mathbf{TD})| + \text{tr}(\mathbf{TD})] [1 + f_t]^{-1} \quad (5)$$

For  $\text{tr}(\mathbf{TD}) < 0$ , the parameter  $A$  will increase, otherwise  $A$  will decrease.

Braun [1997] modified the time history  $f_m$  by using the energy criteria of Chen and Mizuno [1990] instead of the displacement capacity. The energy concentration depends on

$$d\Omega = \varepsilon_{ij} d\sigma_{ij} \quad (6)$$

and the modified history function  $f_m$  is now given by:

$$f_m = \mu_1 \frac{A}{A_{max}} \frac{|d\Omega| + d\Omega}{\|\mathbf{dT}\| \|\hat{\mathbf{D}}\|} \quad (7)$$

with

$$\|\mathbf{dT}\| = \sqrt{d\sigma_{11}^2 + d\sigma_{22}^2 + d\sigma_{33}^2 + 2(d\sigma_{12}^2 + d\sigma_{13}^2 + d\sigma_{23}^2)} \quad (8)$$

$$\|\hat{\mathbf{D}}\| = \sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2 + 2(\varepsilon_{12}^2 + \varepsilon_{13}^2 + \varepsilon_{23}^2)} \quad (9)$$

Analogously to Eq. (5), the increment of the history parameter  $A$  is given by

$$\dot{A} = \mu_2 [|\mathbf{d}\Omega| - d\Omega] - \mu_3 \frac{A}{A_{max}} [|\mathbf{d}\Omega| + d\Omega] \quad (10)$$

### Intergranular strain

A new method to describe the cyclic behaviour of granular material termed the intergranular strain has been developed by Niemunis and Herle. The hypoplastic material laws can properly simulate the material behaviour; however, they are not able to simulate small load amplitudes in the case of cycling loading so that in the hypoplastic material law the deformations are accumulated unrealistically, causing the so-called ratcheting effect.

Both the deformation of the granular skeleton due to grain rearrangement as well as the deformation of the contact area of the bulk material are considered by the intergranular strain. In the case of small cyclic loading prior to the granular skeleton being rearranged, the loading path has to pass through the state of deforming the contact area between the grains, thus reproducing the material memory of the cyclic loaded grain.

The material memory of the granular material is activated only during the period during which the slope of the strain path stays below the stiffness of a monotonic loading. When this stiffness is reached, the material memory of the granular material is

„erased“.

Niemunis et al. [1997] suggested the following expression for the calculation of the stiffness  $M_i$

$$M_i = [\rho^x m_T + (1 - \rho^x) m_R] L + \begin{cases} \rho^x (1 - m_T) L : \hat{\delta} \hat{\delta} + \rho^x N \hat{\delta} & \text{für } \hat{\delta} : D > 0 \\ \rho^x (m_R - m_T) L : \hat{\delta} \hat{\delta} & \text{für } \hat{\delta} : D \leq 0 \end{cases} \quad (11)$$

where  $m_T$ ,  $m_R$ ,  $r$  and  $\chi$  are material parameters.

$\hat{\delta}$  describes the direction of the intergranular strain

$$\hat{\delta} = \begin{cases} \frac{\delta}{\|\delta\|} & \text{for } \delta \neq 0 \\ 0 & \text{for } \delta = 0 \end{cases} \quad (12)$$

and the factor  $\rho$  is given by

$$\rho = \frac{\|\delta\|}{R} \quad (13)$$

The evolution of the intergranular strain can be calculated by:

$$\hat{\delta} = \begin{cases} (1 - \hat{\delta} \hat{\delta} \rho^{\beta_r}) : D & \text{for } \hat{\delta} : D > 0 \\ D & \text{for } \hat{\delta} : D \leq 0 \end{cases} \quad (14)$$

## INTERFACE ELEMENT

The interface element used is based on an interface element described by Ladewig [1994]. It is used to simulate the interaction between the silo shell and the granular material. To neglect or to oversimplify the contact area can total change the results.

The contact area is modelled using an 18-node volume element consisting of two layers of nodes, with the element linking the adjacent nodes of the shell element and the granular material. This correspondence of the two nodes permits a local decoupling of the granular material from the shell. The Mohr-Coulomb law has been used as a friction law.

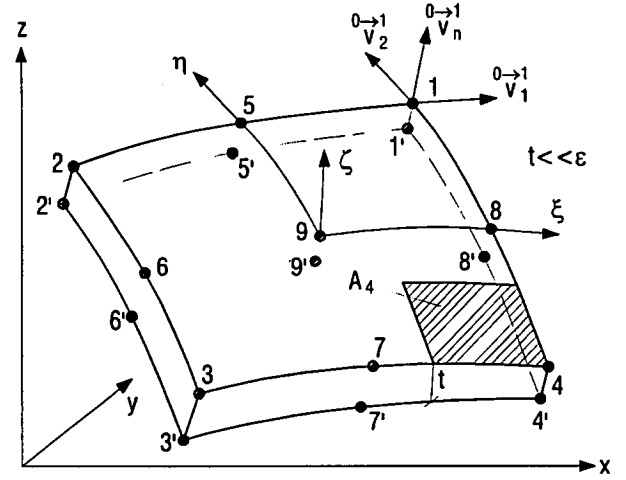


Figure 2: Interface element

The stiffness-matrix consists of spring stiffness values connecting the node  $k$  to the node  $k'$  on the opposite side. To calculate the stiffness matrix, a surface integral has to be evaluated, which gives the corresponding area of each node as part of the overall area. The corresponding tributary area of node  $k$  results from:

$$A_k = \int_A N_k(\xi, \eta) dA = \int_{\xi, \eta} N_k(\xi, \eta) |J_A| d\xi d\eta \quad (15)$$

The spring stiffness in the normal direction  $c_n^k$  is a product of a constant term  $c_d$  and the corresponding area  $A_k$ .

$$c_n^k = c_d A_k \quad \text{for } \Delta u_n^k \leq 0 \quad (16)$$

$$c_n^k = 0 \quad \text{for } \Delta u_n^k > 0 \quad (17)$$

The stiffness in the tangential direction is evaluated in an analogous manner to the stiffness in the normal direction; it is iteratively approximated by the Mohr-Coulomb friction law.

A more detailed explanation can be found in Wagner et al. [2000].

## CYCLIC SOIL MECHANICS TESTS

The simulation of soil mechanics tests helps to estimate the performance of the models employed in correctly describing the cyclic behaviour of the soil materials under investigation. To this purpose, triaxial and oedometric tests under cyclic loading have been chosen, with variable load amplitudes.

The oedometric test shows that the original version of the hypoplasticity theory by von Wolffersdorff is not in a position to simulate the dynamic behaviour of the granular material. This version of hypoplasticity theory accumulates more strains than the three other versions. The material behaviour in reloading is identical to the first loading behaviour (Fig. 3), the typical hardening effect did not appear in the numerical simulation.

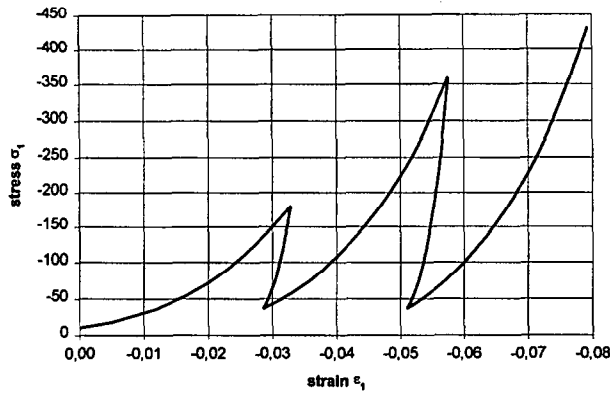


Figure 3: Hypoplasticity simulated oedometric test; stress-strain path

The hypoplasticity theories which have been modified with time history functions and the intergranular strain approach are more appropriate for simulating the cyclic behaviour of the granular material. The hardening effect in the case of reloading is accounted for and the energy dissipation can be described (Fig. 4 and 5).

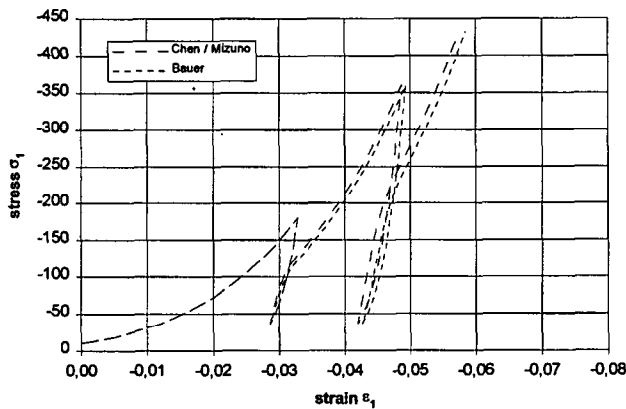


Figure 4: Stress-strain paths of oedometric tests; hypoplasticity theory including time history functions

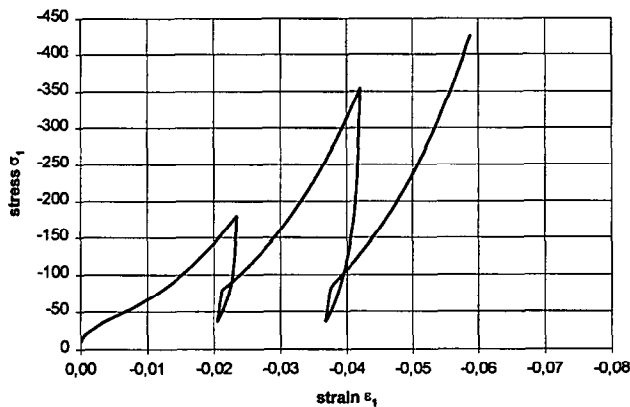


Figure 5: Stress-strain paths of an oedometric tests; intergranular strain approach

As a second experiment, a triaxial test had been investigated numerically. Results showed that both the hypoplasticity theory and the modified version of Bauer were unsuitable for simulating this experiment. The displacement capacity introduced by Bauer cannot be used as a criterion for selecting the correct branch when the load direction changes. On the other hand, the hypoplasticity theory modified with the energy criterion after eq. (7) is able to distinguish between loading and reloading. The stress-strain path is here described by a hysteresis loop and is tantamount to an asymptotical approach in the case of monotonically increasing loads (Fig. 6).

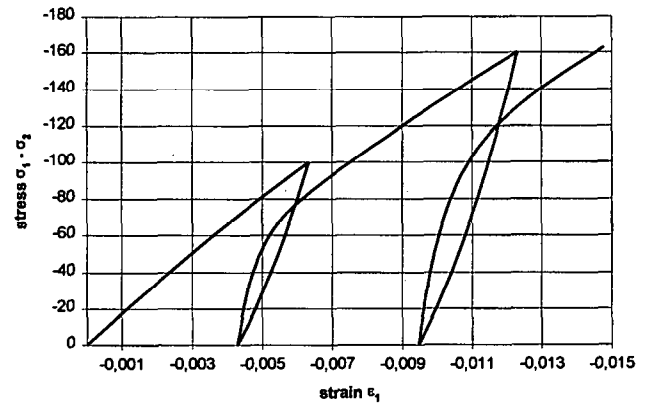


Figure 6: Simulation of the stress-strain path of a triaxial test, by hypoplasticity modified with the energy criterion

The intergranular strain approach can also simulate the cyclic loaded triaxial test.

The results of these tests show that the hypoplasticity theory modified by the energy criterion and the intergranular strain approach are in a position to model cyclic behaviour adequately. The difference between the two material descriptions is shown in a small-strain test. The experimental set-up is identical to the oedometric test apparatus except that the load change is small. The results for two different time-load functions are shown in Fig. 7.

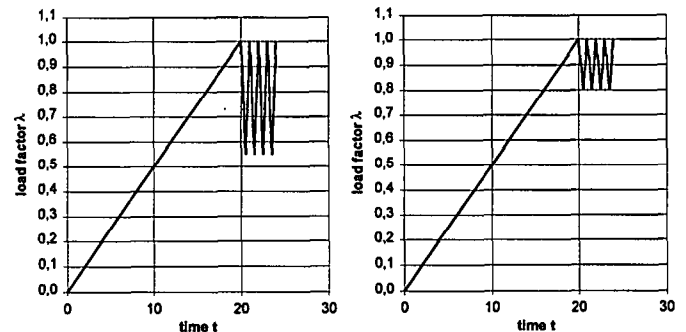


Figure 7: Time-load functions for small strain tests

The hysteresis loops of the hypoplastic material law which has been modified by the time history function show the influence of the function  $f_m$  on limiting the compressibility. The

intersection points of the hysteresis loops move towards the area of higher stresses (Fig. 8).

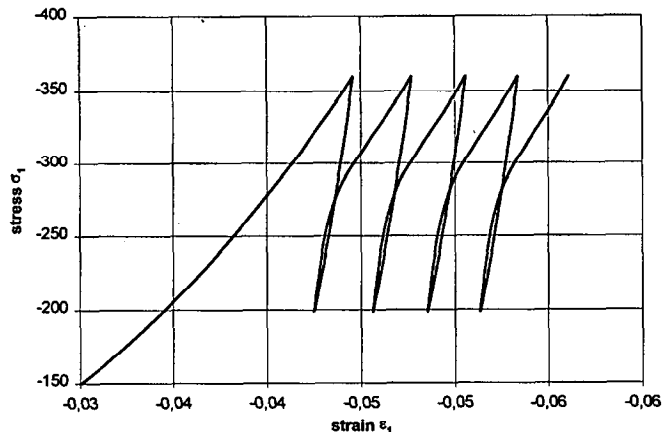


Figure 8: Stress-strain path of the small-strain test; hypoplasticity theory modified with the energy criterion

In hypoplasticity theory, more strain is accumulated than according to the intergranular strain approach. The intergranular strain approach also behaves less dissipatively. A nearly elastic material behaviour for small amplitudes is correctly predicted by the intergranular strain approach (Fig. 9 and 10).

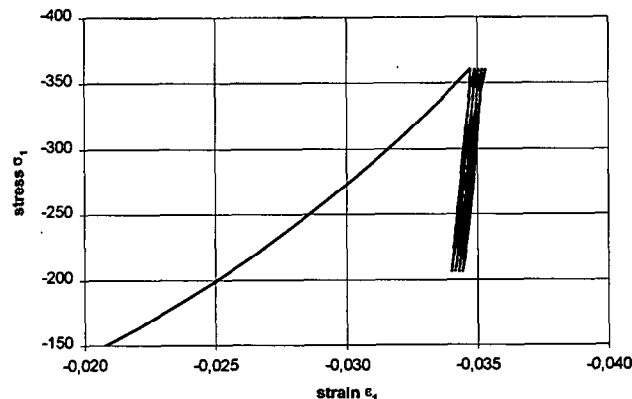


Figure 9: Small-strain test

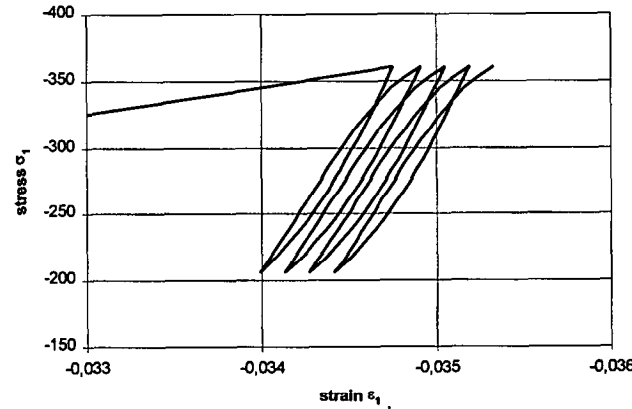


Figure 10: Blow-up region of the small-strain test

The test with reduced amplitudes shows this effect explicitly. Only a blow-up of the region around the reversal point shows that the material did not behave elastically (Figs. 11 and 12). The problem with the ratcheting effect seems to be solved or at least under control.

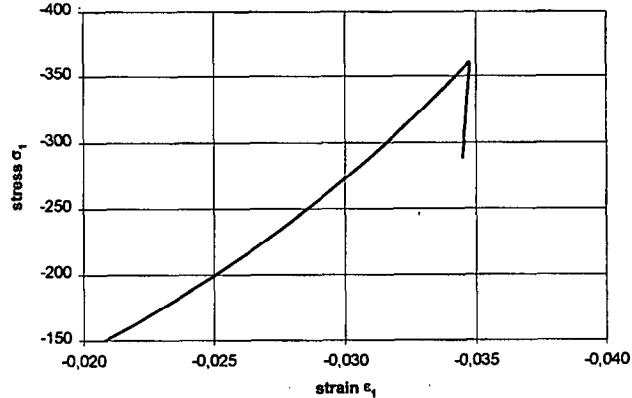


Figure 11: Small-strain test with reduced amplitudes

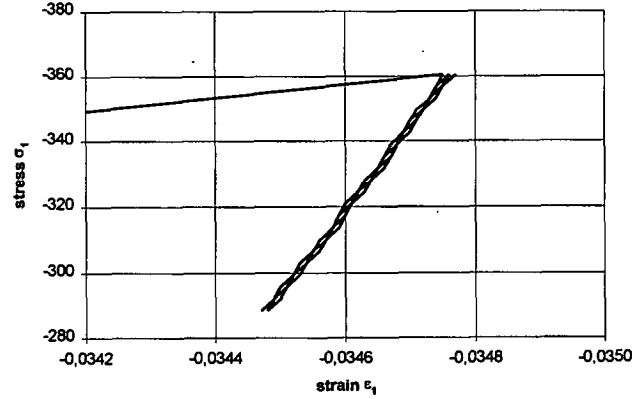


Figure 12: Blow-up region of the small-strain test with reduced amplitudes

### PRESSURE IN SILOS

Having investigated the dynamic behaviour of the material laws, it now remains to be seen if they can be used for modelling silos filled with bulk material. For this purpose, some experiments of silos filled with granular material have been simulated with a numerical model.

As an example, the experimental results by Schütz [1982] will be compared with the results of the numerical simulation using these material laws. The silo of the experiments of Schütz had a height of 6.00 m and a diameter of 0.69 m. The material contained in the silo was sand with a void-ratio of 0.72 and a weight of 15 KN/m³. The frictional coefficient  $\mu$  was 0.60. The charging process has been simulated by monotonically increasing the dead load.

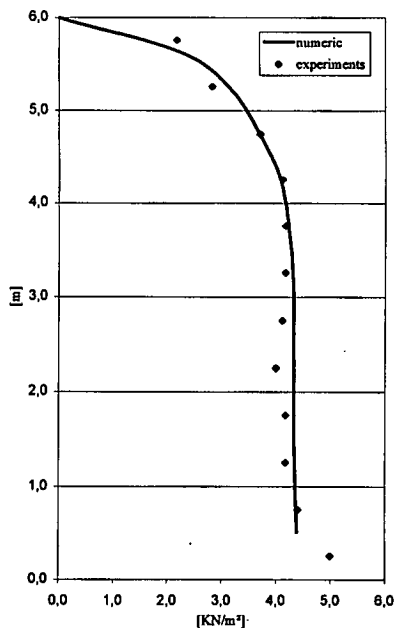


Figure 13: Comparison of the horizontal pressure results of the numerical simulation and the experiment

The comparison of measured and predicted horizontal pressure values as shown in Fig. 13 is quite satisfactory.

## CONCLUSIONS AND FURTHER RESEARCH

The comparison of the different descriptions for the cyclic behaviour of granular material shows that the basic version of the hypoplasticity theory is not able to describe the cyclic behaviour satisfactorily. The modified hypoplasticity theory is able to simulate cyclic oedometric tests, while the modified version with an energy criterion based material law is able to simulate triaxial tests as well. The intergranular strain approach yields the most effective material law. Small-strain tests show that the ratcheting effect can be controlled. The simulation of silos subject to dead load shows that the numerical model used is able to dependably simulate the experimental results.

The next step will be the simulation of silos under earthquake excitation utilizing the two material laws best suited to the task (intergranular approach and hypoplasticity theory modified by the energy criterion). Some corresponding experiments of silos subject to seismic-type loads are currently under way.

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